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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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AIR ADMIXTURE TO EXHAUST JETS

By E. Sänger

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AIR ADMIXTURE TO EXHAUST JETS*

By E. Sänger

1. Introduction.- The customary jet engines - rockets, turbojet engines, pulse jet engines - show at actual flight speeds certain defects, like low efficiencies, failure in case of increased flight velocities, etc. Furthermore, all these power plants have, with the customary propeller-piston power plants, the characteristic in common that they discharge considerable heat quantities with the exhaust gases without utilizing them.

The ram-jet engine has no static thrust, and, in the subsonic region, its thrust increases approximately with the square of the flight velocity; thus, one is forced, in many cases, to combinations with other jet engines for take-off and slow flight.

This situation gives rise to the theoretical investigation of how far such disadvantages can be fundamentally reduced by air admixture to the exhaust jet in special shrouds.

The problem of thrust increase for jet engines by air admixture to the exhaust jet was introduced into aviation techniques by the suggestions of Mélot (ref. 1). Due to a too general interpretation of several theoretical investigations of A. Busemann (ref. 2), so far no practical use has been made of these suggestions.

The following considerations show that, in the case of low-pressure mixing according to Mélot's suggestions, probably no thrust increase of technical significance will occur for the flight speeds of interest (however, the low-pressure mixture is highly promising for ground test setups and for special power plants of relatively slow sea and land vehicles).

In contrast, application of the high-pressure mixing in ram-jet type shrouds, where surrounding air is admixed to the exhaust jet, appears advantageous throughout for some aeronautical power plants in the range of high subsonic and supersonic speeds.

The relative increases become larger the higher the flight speed and the less satisfactory the thermic efficiency of the jet engine used.

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*"Luftzumischung zu Abgasstrahlen," Ingenieur-Archiv, vol. 18, 1950, pp. 310-323.

Fundamentally, the four energy components of two gas jets mix in such a manner that, for constant total energy, the sum of the enthalpies increases at the expense of the sum of the kinetic energies, with the total entropy increasing as well.

If the mixing of the jet takes place at a pressure exceeding the undisturbed external pressure, part of the enthalpy developed may be reconverted into kinetic energy (by the following expansion of the gas mixture) and thus be made useful.

These conditions lead to combination of the ram-jet engine with other power plants in the form of ram-jet type shrouds and mixing chambers, with the purpose of utilizing part of the otherwise completely lost exhaust gas energies, especially the heat energies of the core jets for reheating of an additional ram-jet engine.

Since, in this kind of heating, kinetic energy is unadvoidably supplied beside the thermic energy, we call this principle, in short, impulse heating of the ram-jet engine.

For simplification of the following theoretical investigations, we assume in the calculation that the mixing of the jet always takes place at constant pressure; in practice, however, mixing at variable pressure is applicable as well, particularly in low-pressure chambers for pressure increase, in high-pressure chambers for pressure drop.

I. Constant-Pressure Mixing of Gas Jets

2. Theory of constant-pressure mixing.- A gas jet leaves an opening of cross section F_0 with the velocity v_0 and the remaining state points p, ρ_0, T_0. It mixes with the free surrounding air which flows in the same direction with the velocity v_2 and possesses the remaining state points p, ρ_2, T_2 (fig. 1).

The following relations are valid for the mixing of a certain air mass $m_2 = \rho_2 F_2 v_2$ flowing per second through the cross section F_2 with the corresponding exhaust gas mass $m_0 = \rho_0 F_0 v_0$:

Continuity theorem: The sum of the masses flowing per second through the cross sections F_2 and F_0 equals the mass flowing per second through the cross section F_x

$$\rho_0 F_0 v_0 + \rho_2 F_2 v_2 = \rho_x F_x v_x \tag{1}$$

Momentum theorem: Due to the pressure being constant throughout, the total momentum remains the same in every cross section

$$\rho_0 F_0 v_0^2 + \rho_2 F_2 v_2^2 = \rho_x F_x v_x^2 \tag{2}$$

Energy theorem: The sum of the energies flowing per second through the cross sections F_2 and F_0 equals the energy flowing per second through the cross section F_x with presupposition of a conversion of kinetic energy into enthalpy free from relaxation as in the calculation of the gas throttling

$$\rho_0 F_0 v_0 \left(\frac{v_0^2}{2} + gc_p T_0\right) + \rho_2 F_2 v_2 \left(\frac{v_2^2}{2} + gc_p T_2\right) = \rho_x F_x v_x \left(\frac{v_x^2}{2} + gc_p T_x\right) \tag{3}$$

Equation of state: The values of the quantities ρ_x, T_x are related to the known constant mixing pressure

$$p = g\rho_x R T_x \tag{4}$$

In the jet core, the customary rectangular distribution of the state quantities over the cross section with the same mass flow as the actual distribution was assumed. The gas constants R and c_p are assumed approximately equal for the two gases to be mixed. The turbulence dissipation into enthalpy is assumed to be free from inertia as was said before.

With these four equations, the four unknowns F_x, ρ_x, v_x, T_x may be expressed by the known quantities F_2, ρ_2, v_2, T_2, F_0, ρ_0, v_0, T_0.

For simplification of the notation, new symbols are introduced for the known quantities

Mass sum $\rho_0 F_0 v_0 + \rho_2 F_2 v_2 = M \ [\text{kgsec/m}]$ (5)

Momentum sum $\rho_0 F_0 v_0^2 + \rho_2 F_2 v_2^2 = J \ [\text{kg}]$ (6)

Energy sum $\rho_0 F_0 v_0 \left(v_0^2/2 + gc_p T_0\right) + \rho_2 F_2 v_2 \left(v_2^2/2 + gc_p T_2\right) = E \ [\text{kgm/sec}]$ (7)

Temperature density $p/gR = \rho_0 T_0 = \rho_2 T_2 = \rho_x T_x = D \ [\text{kgsec}^2 \ {}^\circ/\text{m}^4]$ (8)

Therewith the four determining equations become

$$\rho_x F_x v_x = M$$

$$\rho_x F_x v_x^2 = J$$

$$\rho_x F_x v_x \left(\frac{v_x^2}{2} + gc_p T_x\right) = E$$

$$\rho_x T_x = D$$

From them, there follow the unknowns

$$v_x = \frac{J}{M} \tag{9}$$

$$F_x = \frac{ME - \frac{1}{2} J^2}{gc_p DJ} \tag{10}$$

$$\rho_x = \frac{gc_p M^2 D}{ME - \frac{1}{2} J^2} \tag{11}$$

$$T_x = \frac{ME - \frac{1}{2} J^2}{gc_p M^2} \tag{12}$$

and the mixture Mach number

$$\frac{v_x}{a_x} = \sqrt{\frac{2}{\chi - 1} \frac{1}{2ME/J^2 - 1}} \tag{12a}$$

used later on.

3. Propulsive mixing efficiency.- The ratio of the kinetic energy flowing off per second through F_x and the total kinetic energy which has flowed in is

$$\eta_m = \frac{\rho_x F_x v_x^3}{\rho_0 F_0 v_0^3 + \rho_2 F_2 v_2^3} = \frac{J^2/M}{2E - 2gc_p\left(\rho_0 F_0 v_0 T_0 + \rho_2 F_2 v_2 T_2\right)}$$

$$= \frac{\left(1 + \frac{m_2 v_2}{m_0 v_0}\right)^2}{\left(1 + \frac{m_2}{m_0}\right)\left(1 + \frac{m_2 v_2^2}{m_0 v_0^2}\right)} \tag{13}$$

therefore, it is only a function of the two ratios m_2/m_0 and v_2/v_0.

The propulsive mixing efficiency depends neither on the pressure p at which the mixing takes place, nor on the densities, temperatures, enthalpies, or Mach numbers of the mixing partners concerned or on the relaxation of the vortex conversion into heat. The kinetic energies disappearing in the mixing are converted via vortex motion into additional enthalpy as in the process of gas throttling.

The mixing efficiency for $m_2/m_0 \rightarrow 0$ and $m_2/m_0 \rightarrow \infty$ becomes of course equal, $\eta_m = 1$, since, when one of the mixing partners disappears, a mixing, and therefore mixing losses, are no longer possible.

The mixing efficiency becomes for $v_2/v_0 = 0$, that is, mixing with surrounding air at rest, equal $\eta_m = 1/\left(1 + m_2/m_0\right) = m_0/m_x$ thus for mixing ratios in practical use very small; the jet energy is converted almost entirely into vortices and heat.

The mixing efficiency becomes, furthermore, with $v_2/v_0 = 1$ equal to $\eta_m = 1$, that is, when both gas jets have equal velocity of the same direction, the mixing occurs by diffusion without losses in kinetic energy, even when the temperatures of the two jets are different.

In the range of arbitrary values of m_2/m_0 and v_2/v_0, the mixing efficiencies show minima; these minima lie at values of m_2/m_0 the higher, the smaller v_2/v_0; their course may be calculated to be $m_2/m_0 = v_0/v_2$, that is, the losses become largest when both jet impulses are at first equal. The course can be seen in figure 2.

If the gases to be admixed have a priori noticeable velocities in the direction of the gas jet, the mixing efficiencies are throughout considerable and the higher, the more gases are admixed inasmuch as $m_2/m_0 > v_0/v_2$. Only for very small v_2/v_0 or m_2/m_0 this rule is inverted, that is, the efficiency then deteriorates with increasing admixture until $m_0v_0 = m_2v_2$ and rises again afterwards.

Since, in case of mixing efficiences below unity, warming of the gas mixture occurs also when the temperature of both jets is equal, the supply of momentum always has a heating effect as well.

For the mixture Mach number

$$\frac{1 + \frac{2}{\chi - 1}\frac{a_x^2}{v_x^2}}{1 + \frac{2}{\chi - 1}\frac{a_2^2}{v_2^2}} = \frac{1 + \frac{m_2}{m_0}}{\left(1 + \frac{m_2 v_2}{m_0 v_0}\right)^2}\left(\frac{m_2 v_2^2}{m_0 v_0^2} + \frac{1 + \frac{2}{\chi - 1}\frac{a_0^2}{v_0^2}}{1 + \frac{2}{\chi - 1}\frac{a_2^2}{v_2^2}}\right)$$

$$= \frac{1}{\eta_m}\,\frac{\frac{m_2 v_2^2}{m_0 v_0^2} + \frac{1 + \frac{2}{\chi - 1}\frac{a_0^2}{v_0^2}}{1 + \frac{2}{\chi - 1}\frac{a_2^2}{v_2^2}}}{\frac{m_2 v_2^2}{m_0 v_0^2} + 1} \tag{12b}$$

is valid. The mixture Mach number is therefore a function of the mass-, velocity-, and Mach number ratios of the components. When the Mach number ratio of the components becomes unity, the ratio defined above of the air Mach number expression and of the mixture Mach number expression is equal to the mixing efficiency.

4. Discussion of special cases.- The discussion of the limiting cases of the mixing efficiency may be extended to the remaining properties of the mixed jet and leads to remarkable characteristics.

The special case (frequently occurring in practice) $v_2/v_0 = 0$, that is, admixture gas at rest gives because of the disappearing velocity component parallel to the axis of the admixed gas directly $F_2 = \infty$, since continuity and momentum theorem can be satisfied simultaneously only when mixing occurs, thus $\rho_x F_x v_x \neq \rho_0 F_0 v_0$. One may now consider F_x as an arbitrarily selectable independent variable and assume it to be prescribed since to every arbitrary F_x there pertains a $F_2 = \infty$.

From the equations (1) to (4) there follows the quantity (undetermined at first)

$$\frac{F_2 v_2}{F_0 v_0} = \frac{1}{2}\left[\sqrt{\left(\frac{\chi - 1}{2}\frac{v_0^2}{a_0^2} + \frac{T_2}{T_0} + 1\right)^2 + 4\frac{T_2}{T_0}\left(\frac{F_x}{F_0} - 1\right)} - \left(\frac{\chi - 1}{2}\frac{v_0^2}{a_0^2} + \frac{T_2}{T_0} + 1\right)\right] \tag{14}$$

and therewith directly, from the equations (9) and (11) and (12), the unknown quantities v_x, ρ_x, T_x.

A still more restricted special case not infrequent in practice is $v_2/v_0 = 0$ and $T_2/T_0 = 1$. Due to the throughout equal pressures p, one then has also $\rho_2/\rho_0 = 1$, and the undetermined quantity F_2v_2 becomes

$$\frac{F_2v_2}{F_0v_0} = \sqrt{\left(\frac{\chi - 1}{4}\frac{v_0^2}{a_0^2} + 1\right)^2 + \left(\frac{F_x}{F_0} - 1\right)} - \left(\frac{\chi - 1}{4}\frac{v_0^2}{a_0^2} + 1\right) \tag{14a}$$

and hence furthermore the unknowns from equations (9), (11), and (12)

$$\frac{v_x}{v_0} = \frac{1}{1 + F_2v_2/F_0v_0}, \qquad \frac{T_x}{T_0} = 1 + \frac{\chi - 1}{2}\frac{v_0^2}{a_0^2}\frac{\frac{F_2v_2}{F_0v_0}}{\left(1 + \frac{F_2v_2}{F_0v_0}\right)^2}, \qquad \frac{\rho_x}{\rho} = \frac{T}{T_x}$$

In the mixed jet temperature and density are, therefore, completely different from the values of the gases (all equal) before the mixing; the jet mixing has a heating effect as would be shown by a schlieren-optical observation.

A third special case results with $v_2 = v_0$

$$M = v_0\left(\rho_0F_0 + \rho_2F_2\right)$$

$$J = v_0^2\left(\rho_0F_0 + \rho_2F_2\right)$$

$$E = v_0^2\left[\rho_0F_0\left(\frac{v_0^2}{2} + gc_pT_0\right) + \rho_2F_2\left(\frac{v_2^2}{2} + gc_pT_2\right)\right]$$

$$D = \rho_0T_0 = \rho_2T_2$$

and with the aid of equations (9) to (12)

$v_x = v_0$, the flow velocity remains the same after the mixing

$F_x = F_0 + F_2$, the flow cross sections add

$\rho_x = \dfrac{\rho_0F_0 + \rho_2F_2}{F_0 + F_2}$, the densities mix in proportion to the masses

$T_x = \dfrac{\rho_0F_0T_0 + \rho_2F_2T_2}{\rho_0F_0 + \rho_2F_2}$, the temperatures mix in proportion to the enthalpies

Similarly, all other special cases of the gas jet mixture may be derived from section 2.

5. The constant-pressure mixing chamber.- The free jet mixing treated in sections 2 and 3 takes place in the same manner in a closed mixing chamber, if the walls of the chamber have the shape of the streamlines of the admixed gas indicated in figure 1. In this case, the mixing ratio m_2/m_0 also may be arbitrarily limited. Due to the pressure in the admixed gas remaining constant, the flow velocity, temperature, and density of that gas remain constant in the mixing chamber before the mixture is achieved.

The individual, usually decreasing cross-sectional areas F of the mixing chamber may therefore easily be calculated from the decreasing quantity of the admixed gas. At the point on the mixing-chamber axis where the mixing cross section is determined by

$$F_{\xi} = \frac{M_{(\xi)}E_{(\xi)} - \frac{1}{2} J_{\xi}^2}{gc_pDJ_{(\xi)}}$$

the mixing-chamber cross section is, according to the continuity theorem

$$F = F_{(\xi)} + F_{2(x)} - F_{2(\xi)} \tag{15}$$

one uses therein the designations of figure 3 and the symbols have the same significance as in section 2.

The independent parameter $F_{2(\xi)}$ may be selected arbitrarily between F_0 and $F_{2(x)}$ and results in each case in a mixing-chamber cross section F.

Whereas thus the mixing-chamber cross sections may be calculated simply and unequivocally, the actual meridian form, that is, the coordination of these cross sections to given points along the axis of the device, is determinable only on the basis of empirical experiences regarding the actual opening angle of the mixed jet, the meridian form of the mixed jet, the velocity distribution in it, the rate of the turbulence dissipation, etc.

Without knowledge of the results of such tests, one may consider that the velocity distribution in a conical jet with a total opening angle of about 10° to 14° will be homogeneous.

Another important research problem concerning the mixing chamber arises in the use of very hot core jets as are given off for instance from rockets and where the very strong thermal dissociation, which may contain in latent form more than half of the energy supplied to the core jet, is reduced in the mixing with surrounding air and thereby will, in addition, very greatly heat the mixed jet.

The question how far the mixing can be accelerated by special guide vanes in the mixed jet would have to be clarified separately.

As mentioned before, it was further presupposed in the present consideration that the kinetic energy first converted into vortex energy in the jet mixing is further converted, still within the mixing chamber, practically completely into heat. This process, essentially caused by internal friction, is greatly accelerated by the large differences in velocity existing, the high viscosity of the hot combustion gases, and the chemical reactions taking place simultaneously. Thus, it appears justified to calculate as in the customary gas throttling; experimental confirmation, however, is still lacking.

As the examples completely calculated later on show, this assumption is unfavorable for low-pressure mixing chambers, of slight influence for high-pressure mixing chambers which are heated by power plants of small inner efficiency, and favorable for high-pressure mixing chambers heated by power plants of high internal efficiency.

II. Thrust Increase of Jet Engines by Admixture of Air to the Exhaust Jet

6. Theory of thrust increase.- One has

Core thrust: $P_0 = \rho_0 F_0 v_0^2 - \rho_2 F_{20} v_2^2$

Total thrust: $P = \rho_4 F_4 v_4^2 - \rho F v^2$

Factor of thrust distribution: $\dfrac{P}{P_0} = \dfrac{\rho_4 F_4 v_4^2 - \rho F v^2}{\rho_0 F_0 v_0^2 - \rho_2 F_{20} v^2}$

Of the characteristic parameters listed in figure 4, 14 are unknown

$$
\begin{array}{ll}
F, & v_2, \rho_2, T_2 \\
F_3, p_3, & v_3, \rho_3, T_3 \\
F_4, p_4, & v_4, \rho_4, T_4
\end{array}
$$

The parameter p_2 is assumed to be known since it is, by design assumptions, a priori arbitrarily selectable within certain limits.

Of flow equations, there are available:

Zone $F - F_2$: Continuity and energy theorem, adiabatic equation and gas equation

Zone $F_2 - F_3$: See section 2; for the F_2 there one has to put here $F_2 - F_{20}$; use is made of equations (9), (10), (11), (12), and of the relation for constant-pressure mixing $p_3 = p_2$

Zone $F_3 - F_4$: Continuity and energy theorem, adiabatic equation and gas equation, and the pressure reduction condition $p_4 = p$

Thus one has, for the 14 unknowns, an equal number of determining equations. The unknown quantities ρ_4, v_4, F_4, v_2 appearing in the thrust factor P/P_0 are to be calculated from the prescribed quantities p_2, p, v, ρ, T, F_0, p_0, v_0, ρ_0, T_0, F_{20}, F_2.

From the flow equations of the first zone there follow the relations

$$\rho_2 = \rho\left(\frac{p_2}{p}\right)^{\frac{1}{\chi}} \tag{16}$$

$$T_2 = \frac{p_2}{\rho g R}\left(\frac{p}{p_2}\right)^{\frac{1}{\chi}} \tag{17}$$

$$v_2 = \sqrt{v^2 + 2gc_pT\left[1 - \left(\frac{p_2}{p}\right)^{\frac{X-1}{X}}\right]} \tag{18}$$

$$F = F_2\left(\frac{p_2}{p}\right)^{\frac{1}{X}}\sqrt{1 + \frac{2gc_pT}{v^2}\left[1 - \left(\frac{p_2}{p}\right)^{\frac{X-1}{X}}\right]} \tag{19}$$

where the symbols, defined as parameters for equations (5) to (8) become

Mass sum: $$M = \rho_0 F_0 v_0 + \rho(F_2 - F_{20})\left(\frac{p_2}{p}\right)^{\frac{1}{X}}\sqrt{v^2 + 2gc_pT\left[1 - \left(\frac{p_2}{p}\right)^{\frac{X-1}{X}}\right]}\ [\text{kgsec/m}] \tag{5a}$$

Momentum sum: $$J = \rho_0 F_0 v_0^2 + \rho(F_2 - F_{20})\left(\frac{p_2}{p}\right)^{\frac{1}{X}}\left\{v^2 + 2gc_pT\left[1 - \left(\frac{p_2}{p}\right)^{\frac{X-1}{X}}\right]\right\}\ [\text{kg}] \tag{6a}$$

Energy sum: $$E = \rho_0 F_0 v_0\left(\frac{v_0^2}{2} + gc_pT_0\right) + \rho(F_2 - F_{20})\left(\frac{p_2}{p}\right)^{\frac{1}{X}}\sqrt{v^2 + 2gc_pT\left[1 - \left(\frac{p_2}{p}\right)^{\frac{X-1}{X}}\right]}\left(\frac{v^2}{2} + gc_pT\right)\ [\text{kgm/sec}] \tag{7a}$$

Temperature density: $$D = \frac{p_2}{gR}\ [\text{kgsec}^2\ {}^{\circ}/\text{m}^4] \tag{8a}$$

and finally the directly used unknowns themselves

$$\rho_4 = gc_p \frac{M^2 D}{ME - \frac{1}{2}J^2}\left(\frac{p}{p_2}\right)^{\frac{1}{\chi}} \left[kgsec^2/m^4\right] \tag{20}$$

$$T_4 = \frac{ME - \frac{1}{2}J^2}{gc_p M^2}\left(\frac{p}{p_2}\right)^{\frac{\chi-1}{\chi}} \left[{}^{o}K\right] \tag{21}$$

$$v_4 = \sqrt{\frac{J^2}{M^2} + 2\,\frac{ME - \frac{1}{2}J^2}{M^2}\left[1 - \left(\frac{p}{p_2}\right)^{\frac{\chi-1}{\chi}}\right]} \left[m/sec\right] \tag{22}$$

$$F_4 = \frac{\left(ME - \frac{1}{2}J^2\right)\left(\frac{p_2}{p}\right)^{\frac{1}{\chi}}}{gc_p D\sqrt{J^2 + 2\left(ME - \frac{1}{2}J^2\right)\left[1 - \left(\frac{p}{p_2}\right)^{\frac{\chi-1}{\chi}}\right]}} \left[m^2\right] \tag{23}$$

Therewith, one may at last write the desired factor of thrust distribution

$$\left.\begin{aligned} \frac{P}{P_0} &= \frac{\rho_4 F_4 v_4^2 - \rho F v^2}{\rho_0 F_0 v_0^2 - \rho_2 F_{20} v_2^2} \\ &= \frac{\sqrt{2ME\left[1 - \left(\frac{p}{p_2}\right)^{\frac{\chi-1}{\chi}}\right] + J^2\left(\frac{p}{p_2}\right)^{\frac{\chi-1}{\chi}}} - \rho F_2 v^2\sqrt{1 + \frac{2gc_p T}{v^2}\left[1 - \left(\frac{p_2}{p}\right)^{\frac{\chi-1}{\chi}}\right]}\left(\frac{p_2}{p}\right)^{\frac{1}{\chi}}}{\rho_0 F_0 v_0^2 - \rho F_{20} v^2\left\{1 + \frac{2gc_p T}{v^2}\left[1 - \left(\frac{p_2}{p}\right)^{\frac{\chi-1}{\chi}}\right]\right\}\left(\frac{p_2}{p}\right)^{\frac{1}{\chi}}} \end{aligned}\right\} \tag{24*}$$

*The NACA reviewer has pointed out that the quantity $(p_2/p)^{1/\chi}$ in the denominator of this equation was erroneously inverted in the German text.

7. Discussion of the thrust increase.- The thrust factor P/P_0 does not immediately signify the thrust increase of a jet engine by addition of the shroud under otherwise equal circumstances. Rather it describes, in the first place, the distribution of the thrust between the core jet and the shroud for the respective state of flight ρ, v.

However, for this state of flight, the core jet without shroud, might have a thrust essentially different from the thrust it would have with a shroud. In practice, it will happen not infrequently that the core-jet thrust without shroud is considerably smaller or even zero so that the actual thrust increase for a certain state of flight by addition of the shroud can be much larger than P/P_0, even infinite. This case occurs for instance for pulse jet tubes and high flight velocities. On the other hand, the thrust of core jets might be reduced by addition of a shroud, as in rockets, although this effect will often remain negligibly small. In the last special case P/P_0 then actually signifies the thrust increase of only the core jet by addition of the shroud.

The physical technical significance of equation (24) will become even clearer by discussion of a few special cases.

(a) Special case $v = 0$, that is, static thrust. Equation (24) is specialized to the form

$$\frac{P}{P_0} = \frac{\sqrt{\dfrac{\rho_0 F_0 v_0}{2g c_p T_0}}}{\dfrac{\chi - 1}{2}\,\dfrac{\rho_0 F_0 v_0^2}{a_0^2} - \dfrac{\rho F_{20} T}{T_0}\left[\left(\dfrac{p_2}{p}\right)^{\frac{1}{\chi}} - \left(\dfrac{p_2}{p}\right)\right]}\left\{\rho_0 F_0 v_0\left[\frac{\chi - 1}{2}\,\frac{v_0^2}{a_0^2} + 1 - \left(\frac{p_2}{p}\right)^{\frac{1-\chi}{\chi}}\right] + \right.$$

$$\rho(F_2 - F_{20})\sqrt{2g c_p T\left[\left(\frac{p_2}{p}\right)^{\frac{2}{\chi}} - \left(\frac{p_2}{p}\right)^{\frac{1+\chi}{\chi}}\right]}\left[\frac{\chi - 1}{2}\,\frac{v_0^2}{a_0^2} + 1 + \frac{T}{T_0}\right]\left[1 - \left(\frac{p_2}{p}\right)^{\frac{1-\chi}{\chi}}\right] +$$

$$\left. 2\rho(F_2 - F_{20})v_0\,\frac{T}{T_0}\left(\frac{p_0}{p}\right)^{\frac{1-\chi}{\chi}}\left[\left(\frac{p_2}{p}\right)^{\frac{1}{\chi}} - \left(\frac{p_2}{p}\right)\right]\right\}^{\frac{1}{2}} \tag{24a}$$

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*NACA reviewer's correction: The erroneous term $\frac{\rho = F_0 v_0^2}{a_0}$ in the denominator of the German text was changed to the correct form $\frac{\rho_0 F_0 v_0^2}{a_0^2}$.

With $p_2/p > 1$ the solution becomes imaginary; this case, as is immediately clear, is physically not realizable. With $p_2/p = 1$ one obtains $P/P_0 = 1$; this case is the same as in the jet mixing with free surrounding air and an increase in thrust does not occur.

With $p_2/p < 1$, real solutions with $P/P_0 > 1$ are possible as long as the sum of the three terms under the large square root sign remains positive. Since the expression

$$\left[1 - (p_2/p)^{\frac{1-\chi}{\chi}}\right]$$

is always negative, the second term will always be negative and the first, too, becomes negative when the enthalpy of the core jet is large compared to its kinetic energy, thus, the Mach number v_0/a_0 is small. One understands immediately, by means of the following consideration, that, for the static case, even for $p_2 < p$, cases may exist where the flow is physically not realizable.

From the elementary gas dynamic relation

$$\frac{p}{p_2} = \left(1 + \frac{v_2^2}{2gc_pT_2}\right)^{\frac{\chi}{\chi-1}} = \left(1 + \frac{\chi - 1}{2}\frac{v_2^2}{a_2^2}\right)^{\frac{\chi}{\chi-1}}$$

there follows that the pressure ratio for the flow either in the static or dynamic case depends only on the Mach number. If there becomes for instance $v_3^2/a_3^2 < v_2^2/a_2^2$, that is, if, in the mixing process, the enthalpy increases more than the kinetic energy, thus decreasing the Mach number, the higher external pressure can no longer be attained at all after the constant-pressure mixing. This case will occur in practice quite frequently in the mixing of hot slow combustion gas jets with cold air, particularly when, in addition, reverse dissociation occurs.

Generally, the static thrust increase will be zero, thus $P = P_0$, when

$$\frac{p}{p_2} \leqq \left(\frac{P_0^2 - 2ME}{J^2 - 2ME}\right)^{\frac{\chi}{\chi-1}} \quad \text{or} \quad v_2 \leqq \sqrt{gc_pT_2\frac{J^2 - P_0^2}{ME - \frac{J^2}{2}}}$$

that is, one will have to work sometimes with very high mixing chamber inflow velocities v_2 in order to attain high mixing efficiencies if the static thrust value is important. The v_2 optimum for static thrust may be immediately determined for any given p_2/p from the equation for P/P_0.

With the aid of equation (12a), one finds generally that, for an efficiency η_D of the end-diffuser different from unity the undisturbed external pressure p is again attainable when

$$\frac{\frac{v_0}{a_0}}{\frac{v_2}{a_2}} \geqq \sqrt{\frac{\frac{2}{\chi - 1}\frac{a_2^2}{v_2^2}}{\frac{\left(\frac{m_2 v_2}{m_0 v_0} + 1\right)^2}{\frac{m_2}{m_0} + 1}\left[\frac{\frac{\chi - 1}{2}\frac{v_2^2}{a_2^2} + 1}{\frac{\chi - 1}{2}\frac{v_2^2}{a_2^2} + 1 - \left(\frac{p - \Delta p}{p - \frac{\Delta p}{\eta_D}}\right)^{\frac{\chi-1}{\chi}}}\right] - \frac{m_2 v_2^2}{m_0 v_0^2}\left(1 + \frac{2}{\chi - 1}\frac{a_2^2}{v_2^2}\right) - 1}} \tag{12c}$$

(b) Special case $v_0 = 0$, that is, momentum less heat supply, with the static thrust becoming immediately zero, as known from the standard ram-jet engine.

The effect of the fuel injection as a core jet will be mostly negligible in this case; that is, the core thrust P_0 becomes zero.

For the pure ram-jet thrust, there then follows with

equation (5a):
$$M = M_0 + \rho F_2\left(\frac{p_2}{p}\right)^{\frac{1}{\chi}}\sqrt{v^2 + 2gc_pT\left[1 - \left(\frac{p_2}{p}\right)^{\frac{\chi-1}{\chi}}\right]}$$

equation (6a):
$$J = \rho F_2\left(\frac{p_2}{p}\right)^{\frac{1}{\chi}}\left\{v^2 + 2gc_pT\left[1 - \left(\frac{p_2}{p}\right)^{\frac{\chi-1}{\chi}}\right]\right\}$$

equation (7a): $E = M_0 gH + \rho F_2\left(\frac{p_2}{p}\right)^{\frac{1}{\chi}}\sqrt{v^2 + 2gc_pT\left[1 - \left(\frac{p_2}{p}\right)^{\frac{\chi-1}{\chi}}\right]}\left(\frac{v^2}{2} + gc_pT\right)$

when M_0 is the mass of the added fuel and H its thermal value in kgm/kg, from equation (24)

$$P = \sqrt{2ME\left[1 - \left(\frac{p}{p_2}\right)^{\frac{\chi-1}{\chi}}\right] + J^2\left(\frac{p}{p_2}\right)^{\frac{\chi-1}{\chi}}} - \rho F_2 v^2\sqrt{1 + \frac{2gc_pT}{v^2}\left[1 - \left(\frac{p_2}{p}\right)^{\frac{\chi-1}{\chi}}\right]}\left(\frac{p_2}{p}\right)^{\frac{1}{\chi}}$$

With the known assumptions $F_2/F \rightarrow \infty$, $M_0 \rightarrow 0$, this formula may be transformed into the simple approximation formula for the thrust of the ram jet given before

$$P = \rho F v^2\left(\sqrt{\frac{T_3}{T_2}} - 1\right)$$

It differs, in addition, by the constant values of the specific heat from the exact calculation of the ram-jet engine according to ZWB-UM 3509 (NACA TM 1106).

(c) Special case $T = T_0$ without further peculiarities.

(d) *Special case $(F_2 - F_{20}) = 0$. With

$$M = \rho_0 F_0 v_0, \qquad J = \rho_0 F_0 v_0^2, \qquad E = \rho_0 F_0 v_0\left(\frac{v_0^2}{2} + gc_pT_0\right)$$

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*NACA reviewer's footnote: The symbol F_{20} was misprinted in the German as F_2.

$\frac{P}{P_0}$ becomes

$$\frac{P}{P_0} = \frac{\rho_0 F_0 v_0^2 \sqrt{1 + \frac{2gc_pT_0}{v_0^2}\left[1 - \left(\frac{p}{p_2}\right)^{\frac{\chi-1}{\chi}}\right]} - \rho F_2 v^2 \sqrt{1 + \frac{2gc_pT}{v^2}\left[1 - \left(\frac{p_2}{p}\right)^{\frac{\chi-1}{\chi}}\right]}\left(\frac{p_2}{p}\right)^{\frac{1}{\chi}}}{\rho_0 F_0 v_0^2 - \rho F_{20} v^2 \left\{1 + \frac{2gc_pT}{v^2}\left[1 - \left(\frac{p_2}{p}\right)^{\frac{\chi-1}{\chi}}\right]\right\}\left(\frac{p_2}{p}\right)^{\frac{1}{\chi}}}$$

Thus after eliminating the admixed air, in general, a thrust increase remains, which follows from the formulation, since the thrust P_0 of the core jet had been referred to the moderate flow velocity within the shroud and additional useful pressure drops originate due to the higher flight velocity. Only with $p = p_0$ one finally has $P/P_0 = 1$, for instance, in case of rockets as core jets, when with $F_{20} = 0$ also $p/p_2 = 1$.

(e) Dependence of the P/P_0 on F_2. In equation (24), in the expression

$$\sqrt{2ME\left[1 - \left(\frac{p}{p_2}\right)^{\frac{\chi-1}{\chi}}\right] + J^2\left(\frac{p}{p_2}\right)^{\frac{\chi-1}{\chi}}}$$

the terms quadratic in F_2 become equal to the second numerator term $\rho F v^2$, and thus disappear for very large F_2 so that P/P_0 then becomes, for very small v/v_0, approximately proportional to $\sqrt{F_2}$. It therefore becomes proportional to the square root of the admixed mass whereas otherwise P/P_0, independently of F_2 and the admixed mass, tends toward a fixed limiting value.

This behavior is known from the elementary theory of the Melot device as well as of the ram-jet device.

Generally, one may, by augmenting F_2, increase the thrust not without limit but only up to a fixed limiting value. Only at the flight speed zero it increases theoretically with $\sqrt{F_2}$ without limit.

(f) Dependence of the P/P_0 on p/p_2. For $p/p_2 = 1$ thrust increase does not occur in any case; equation (24) yields $P/P_0 = 1$. For $p/p_2 < 1$ there originate throughout real values for P/P_0 which are larger than unity, whatever may be the flight speed v, ratios of masses, velocities, or temperatures. Under these circumstances, there exists therefore no range of flight speed or of the other operational conditions where the shroud would lose its thrust-improving effect.

For $p/p_2 > 1$ the situation changes completely. Here, cases with $P/P_0 \gtreqless 1$ as well as cases with $P/P_0 \leqq 0$ are possible. The term chiefly responsible for the thrust

$$\rho_4 F_4 v_4^2 = M\sqrt{\frac{J^2}{M^2} + 2\,\frac{ME - \frac{J^2}{2}}{M^2}\left[1 - \left(\frac{p}{p_2}\right)^{\frac{\chi-1}{\chi}}\right]}$$

contains under the square root sign two energy terms: the total kinetic energy J^2/M^2 present at the end of the mixing chamber F_3 and the addition kinetic energy $2\,\frac{\left(ME - \frac{1}{2}\,J^2\right)}{M^2}$ originating during the process of expansion in the discharge nozzle between F_3 and F_4 from the total enthalpy present in F_3 when $p/p_2 < 1$, or kinetic energy reconverted into enthalpy when $p/p_2 > 1$; thus, compression flow exists as is here to be considered.

When p/p_2 is larger than unity but still so small that in the compression flow so little kinetic energy

$$2\,\frac{ME - \frac{1}{2}\,J^2}{M^2}\left[1 - \left(\frac{p}{p_2}\right)^{\frac{\chi-1}{\chi}}\right]$$

is consumed that there still remains

$$\left(\rho_4 F_4 v_4^2 - \rho F v^2\right) - \left(\rho_0 F_0 v^2 - \rho_2 F_{20} v_2^2\right) > 1$$

we obtain again a positive thrust increase $P/P_0 > 1$.

When p/p_2 has exactly the magnitude that

$$\sqrt{2ME\left[1-\left(\frac{p}{p_2}\right)^{\frac{X-1}{X}}\right]+J^2\left(\frac{p}{p_2}\right)^{\frac{X-1}{X}}} - \rho F_2 v^2\sqrt{1+\frac{2gc_pT}{v^2}\left[1-\left(\frac{p_2}{p}\right)^{\frac{X-1}{X}}\right]} =$$

$$\rho_0 F_0 v_0^2 - \rho F_{20} v^2\left\{1+\frac{2gc_pT}{v^2}\left[1-\left(\frac{p_2}{p}\right)^{\frac{X-1}{X}}\right]\right\}\left(\frac{p_2}{p}\right)^{\frac{1}{X}}$$

the thrust increase disappears, that is, $P/P_0 = 1$.

If p/p_2 further increases, the left side of the above equation becomes smaller than the right side; due to the shroud, there appears immediately a loss in thrust, that is, $P/P_0 < 1$.

If p/p_2 is augmented still further, for instance, until

$$\sqrt{2ME\left[1-\left(\frac{p}{p_2}\right)^{\frac{X-1}{X}}\right]+J_2\left(\frac{p}{p_2}\right)^{\frac{X-1}{X}}} = \rho F_2 v^2\sqrt{1+\frac{2gc_pT}{v^2}\left[1-\left(\frac{p_2}{p}\right)^{\frac{X-1}{X}}\right]}\left(\frac{p_2}{p}\right)^{\frac{1}{X}}$$

is valid, the thrust of the core jet is exactly used up by the processes in the shroud; there remains $P/P_0 = 0$ and the total thrust of the arrangement becomes zero.

All these cases are definitely realizable design, and have probably been actually realized as proved by the numerous failures of related tests.

This holds true also for the case of still larger p/p_2, for instance for the case that exactly all kinetic energy J^2/M^2 present in F_3 is used up during the recompression to the external pressure which is achieved when

$$\frac{p}{p_2} = \left[1+\frac{\frac{J^2}{M^2}}{\frac{2\left(ME\ \frac{1}{2}J^2\right)}{M^2}}\right]^{\frac{X}{X-1}}$$

In spite of all the energy absorption, the entire power plant behaves the same as a pitot tube, has therefore on the whole, only drag.

From these considerations, there results that the jet mixing must take place, if possible, at pressures exceeding the static pressure of the outer air so that expansion to external pressure occurs after the mixing, whereby a compression to external pressure is not necessary. That is, fundamentally, melot type low-pressure mixing nozzles are less favorable or quite useless, and ram-jet type high-pressure mixing nozzles more favorable.

Since low-pressure shrouds therefore promise advantages only for the static case and at very low speeds of motion, we shall deal with them here only by comparison or for ground setups.

(g) Dependence of the P/P_0 on the inner efficiency of the core jet. The ratio between the kinetic jet energy and the total jet energy of the different jet engines, characterized by the jet Mach number varies to an extreme extent.

For rockets, this ratio approaches sometimes 50 percent, for turbojets it lies around 10 percent, and for pulse jet tubes it drops to a few percent. For the basic application of momentum heating to the exhaust gases of these power plants, it is therefore worth knowing how far the attainable thrust increase P/P_0 depends on these inner efficiencies. One may write equation (24) for this purpose in the form

$$\frac{P}{P_0} = \frac{J\sqrt{\frac{2ME}{J^2}\left[1 - \left(\frac{p}{p_2}\right)^{\frac{X-1}{X}}\right] + \left(\frac{p}{p_2}\right)^{\frac{X-1}{X}}} - \rho F_2 v^2\sqrt{1 + \frac{2gc_pT}{v^2}\left[1 - \left(\frac{p_2}{p}\right)^{\frac{X-1}{X}}\right]}\left(\frac{p_2}{p}\right)^{\frac{1}{X}}}{J - \rho F_2 v^2\left\{1 + \frac{2gc_pT}{v^2}\left[1 - \left(\frac{p_2}{p}\right)^{\frac{X-1}{X}}\right]\right\}\left(\frac{p_2}{p}\right)^{\frac{1}{X}}}$$

In many cases which are important in practice, the ratio P/P_0 is determined predominantly by those terms which contain M, E, and J, that is, it will increase with

$$\sqrt{\frac{2ME}{J^2}\left[1 - \left(\frac{p}{p_2}\right)^{\frac{X-1}{X}}\right] + \left(\frac{p}{p_2}\right)^{\frac{X-1}{X}}}$$

Large values of ME/J^2, that is, large ratios between total and kinetic energy or small jet Mach numbers according to equation (12a), however, signify nothing else but low thermal efficiencies.

For otherwise equal conditions, one may therefore expect higher thrust improvements with core jets of low inner efficiency, that is, small jet Mach number (for instance, pulse jet tubes) than for instance with turbojets or rockets.

Since the thrust of the shroud is determined by the thermal energy loss, and the core thrust by the "useful" kinetic energies, the above statement is quite plausible.

The influence of the thermal efficiency frequently is greater than the influence, large in itself, of the mixing-chamber pressure. In practice, the values of ME/J^2 lie approximately between 10 and 40 whereas the values of

$$\left[1 - (p/p_2)^{\frac{\chi-1}{\chi}}\right]$$

(at the high subsonic speeds treated here first) vary between 0.05 and 0.1, and the second term under the square root sign always remains close to unity.

Thus, the differences in the thrust factors may become larger due to the selection of different core jets than due to different mixing chamber pressures.

III. Examples

8. Air admixing shroud heated by rocket exhaust gases.- Let a rocket be prescribed with the jet characteristics $\rho_0 = 0.02$ kgsec^2/m^4, $v_0 = 2000$ m/sec, $F_0 = 0.04$ m^2, $T_0 = 1750°$ K; hence $\rho_0 F_0 v_0 = 1.6$ kgsec/m, $\rho_0 F_0 v^2 = 3200$ kg, $\rho_0 F_0 v_0 \left(v_2^2/2 + gc_pT_0\right) = 6{,}720{,}000$ kgm/sec (= 90000 hp). Let the dissociation heat still bound at the mouth of the rocket be 3,620,000 kgm/sec, corresponding to a lower mixture-thermal value of the rocket propellants of 1540 kcal/kg. The jet Mach number is $v_0/a_0 = 2.63$, the expansion ratio of the rocket $p_1/p_0 = 23$. Favorable arrangements of thrust increasing air-admixing shrouds at flight speeds 0 and 800 km/h are to be indicated.

For $v = 0$ km/h naturally only a low-pressure mixing chamber comes into question. Since drag of the shroud itself here matters little and

the static thrust is known to increase without limit with the admixed masses, an optimum magnitude for this case cannot be stated as is also shown by the differentiation of equation (24) with respect to F_2.

If one chooses arbitrarily a diameter of 2m for the F_2 cross section, and the state of the outer air to correspond to the standard atmosphere, and does not at first assume a value for the mixing chamber pressure p_2 which is likewise arbitrarily selectable, there follow for $F_{20} = 0$ and $v = 0$, the values for M, J, and E as functions of p/p_2 from the equations (5a) to (7a), and the corresponding thrust coefficient from equation (24).

It is known that $P/P_0 = 1$ for $p_2/p = 1$, likewise for small values of p_2/p. Hence, there must necessarily exist an optimum value for p_2/p with respect to thrust increase which may be determined by differentiation of equation (24) with respect to p_2/p to be about 0.7 as also shown by the representation of this equation in figure 5 (without reverse dissociation). P/P_0 there becomes approximately 2.3, that is, the total thrust is increased by the shrouding from 3.2 to 7.3 tons. If the rocket nozzle is adjusted to the new pressure ratios, p/p_0 becomes 33, the exhaust velocity increases to 2080 m/s, and there follows a small additional core jet thrust of 0.13 tons with which the entire calculation would have to be repeated. If the end diffuser of the shroud has an efficiency different from unity, one attains only a slightly higher mixing chamber pressure p_2 which, according to figure 5, has only a small effect on P/P_0 because of the flat thrust maximum.

Corresponding to the pressure drop of 0.7 p there is an inflow velocity of 238 m/s and an inflow Mach number of 0.72. The mixing efficiency becomes about 55 percent and the mixture Mach number, according to equation (12a), remains sufficiently far above the inflow Mach number to insure the reincrease to the external pressure.

With the aid of equations (16) to (23), the remaining desired quantities may be calculated. The dimensions of this shroud are represented in figure 6 and are reminiscent of wind-tunnel proportions. Possibilities of its application for very slow aircraft, launching devices, or water and land vehicles seem therefore dubious. On the other hand, these low-pressure shrouds are interesting for ground test setups, for instance, for investigation of rocket engines operating at low nozzle opening pressures or of entire rocket devices with high approach flow velocities and for subsonic and supersonic wind tunnels of very large test cross section with rocket propulsion, particularly by high-pressure low-temperature (for instance, water vaper) rockets.

In the calculation, so far, the possibility was disregarded that the considerable dissociation of the rocket jet does not stop during the relatively lengthy mixing procedure, in spite of the very considerable cooling of the combustion gases, but does reassociate so that the dissociation energy mentioned at the beginning additionally heats the mixture.

M and J remain, of course, unchanged, whereas E increases correspondingly; hence, the thrust coefficient also varies according to equation (24) in the manner represented in figure 5 (with reverse dissociation).

The optimum of the mixing chamber pressure now lies at $p_2/p = 0.9$ and reduces the thrust quotient optimum there to $P/P_0 = 1.6$; thus, the total thrust then increases from 3.2 tons to only 5.1 tons. The further heat supply by reverse dissociation and possibly afterburning of the combustion gases has therefore, in the low-pressure mixing chambers, a very deteriorating effect on the thrust improvement. In this case, minimizing the vortex conversion into heat would therefore be advantageous.

At a flight speed above zero, one finds, for the shroud, a region with two optimum mixing chamber pressures, one below and one above the external pressure of the air at rest. The first, with increasing v, soon becomes insignificant so that one is concerned only with the high-pressure mixing chambers at all flight speeds of practical interest.

The optimum thrust increases become largest in the static case or for very low flight speeds (region of good Melot effect), pass through a region of very moderate values at medium subsonic speeds, and finally, approaching sonic velocity (region of good Lorin effect), increase again to higher values which, however, remain far behind the high initial values. Only in the supersonic region does the high-pressure shroud become equivalent to the low-pressure shroud in the static case.

In flight, at $v = 800$ km/h, the air drag W of the shroud must be taken into consideration in determining the optimum conditions. One can again express the thrust increase $(P - W)/P_0$ as a function of the quantities to be determined, F_2, and p_2/p, and find their optimum values. For example, for the conditions represented in figure 7 it is found to be $p_2/p = 1.2$, and a thrust increase of about 20 percent is found which corresponds to a thrust coefficient of the shroud of approximately $c_s = 0.2$. Thus with the assumptions made here (full reconversion of the dissociation and turbulence into heat) one finds this thrust increase is 30 percent of the maximum thrust possible from a prescribed ram-jet shroud.

Whereas, in the low-pressure mixing, one could observe expansions of the entrance and exit cross section, the high-pressure mixing chamber showed the known narrowing of these two cross sections.

On the whole, one will conclude from the moderate thrust increases of this example that the momentum heating of the ram-jet engine by rocket exhaust gases problably will have technical significance only for special conditions.

9. Ram-jet tube shroud heated by exhaust gases of turbojet or pulse-jet tubes.- Let the necessary characteristic parameters of the entering and leaving gas jet of a jet engine of moderate jet Mach number at the flight speed of 200 km/h be prescribed. The arrangement of a ram-jet shroud with maximum thrust is desired; the flow velocity at inlet and outlet of the power plant is to remain unchanged, and the flight speed is to be $v = 800$ km/h.

One will choose an arrangement according to figure 4 where the core jet operates within the shroud in the adiabatically decelerated air in a medium of the same surrounding velocity as in the initial state (200 km/h) but with increased values of pressure, density, and temperature. Due to these changes alone, the thrust increases by 15 percent, the fuel consumption by 17 percent.

With consideration of the drag of the shroud, one may again represent, with the aid of equations (5a) to (7a) and (24), the thrust coefficient as a function of the cross section area F_2 of the shroud, and one obtains a flat maximum for instance in the neighborhood of $F_2/F_0 = 2.3$ of $(P - W)/P_0 = 1.2$. The total thrust increases due to the shrouding from the standard value at 200 km/h to a value by 20 percent higher at 800 km/h without noticeable change in the fuel consumption, thus, with a multiple of the total efficiency. The practical importance of this arrangement lies not so much in the moderate thrust increase in itself as in the fact that the increased thrust may still be expected for a flight speed at which the operation of a simple pulse jet tube is altogether questionable. The shroud simulates at 800 km/h flight speed the conditions of 200 km/h which are more favorable for the core jet, particularly the lower diffuser entrance velocity. The internal-pressure level of the core jet increased by almost the whole free-stream impact pressure and therewith provided the compensation of the high additional opening pressures on the air control valve flaps, and improved the air supply from the rear for the frontal ram effect of the air.

Whereas the efficiency of the jet tube mentioned as an example amounted to about 1.5 percent for $v = 200$ km/h, it can be increased by the shrouding, for $v = 800$ km/h, to 8 percent so that it comes quite close to the known efficiencies of simple turbojets and the range of flying missiles thus equipped may become very considerable.

Figure 8 shows the approximate thrust variation against the flight velocity of a pulse jet tube without shroud, with the previously mentioned free shroud, and, finally, with a shroud attached to an existing airplane fuselage or wing in such a manner that no additional drags are caused by it.

The shrouding causes the thrust variation of the pulse jet tube to become similar to that of a turbojet. This circumstance which is very favorable to the pulse jet tube shows simultaneously that the effect of the shrouding on a turbojet remains by far smaller since the standard equipment of the latter anticipates a large part of the effects utilized by the shrouding.

10. Air admixing shroud heated by exhaust gases of piston power plants.- A 4000-hp piston power plant yields for 800 km/h flight speed about 950 kg propeller thrust and approximately 3.8 kg/sec exhaust gases of 600° C temperature. The free-stream impact pressure near the ground is $q = 3080\ \mathrm{kg/m^2}$. If one wanted to mix this quantity of exhaust gas with an air quantity as large as possible, for instance 100 times as large, the required F_1 would be $F_1 = 1.39\mathrm{m}^2$, and the temperature after the mixing would be

$$T_3 = (873 + 100(288 + v^2/2000))/101 = 318^\circ\ \mathrm{K}$$

in order to make the shroud thrust P become

$$P = 2qF_1\left(\sqrt{T_3/T_2} - 1\right) = 81.3\ \mathrm{kg}$$

that is, 8.5 percent of the propeller thrust.

The natural drag of the high-pressure shroud was disregarded as well as that of the entire piston power plant; a possible direct thrust of the exhaust gas jet remains practically unaffected by the additional equipment. For lower flight speeds, this thrust ratio deteriorates, for higher ones, it improves.

IV. Summary

The admixing of surrounding air to exhaust gas jets of power plants may have a thrust-increasing effect, if it takes place at a pressure other than the surrounding pressure.

Admixing in low-pressure mixing chambers, it is true, is limited to such moderate speeds of motion that the flight impact pressure does not yet have too much of a disturbance effect on the low pressure.

In these cases, there result very considerable thrust increases which make the low-pressure mixing interesting for fixed installations in subsonic and supersonic wind tunnels, altitude test chambers, and water or land vehicles.

Admixing in high-pressure chambers, in contrast, becomes the more effective the higher the speed of motion so that this type is suitable especially for aircraft in the high subsonic or in the supersonic range.

The relative thrust increase, under otherwise equal circumstances, is the greater, the smaller the Mach number of the original exhaust gas jet. This increase rises therefore sharply in the following order: rocket, turbojet power plant, pulse jet power plant.

For the last type of power plant, the operation of which is sensitive against high approach flow velocities and low air densities, one may in high-pressure mixing chambers attain an improvement in climate by which the flight speed range and flight altitude range for pulse jet power plants are extended.

One may expect from this type of power plant, under these circumstances and at high flight speeds, total efficiencies which approach those of the turbojet power plants.

The relative thrust increase for low-pressure mixing chambers is greatest in the static case; with increasing speed of motion it drops very rapidly and disappears at fractions of the Mach number 1.

The relative thrust increase for high-pressure mixing chambers is zero in the static case, rises very rapidly with increasing speed of motion, and attains maximum values in the supersonic range.

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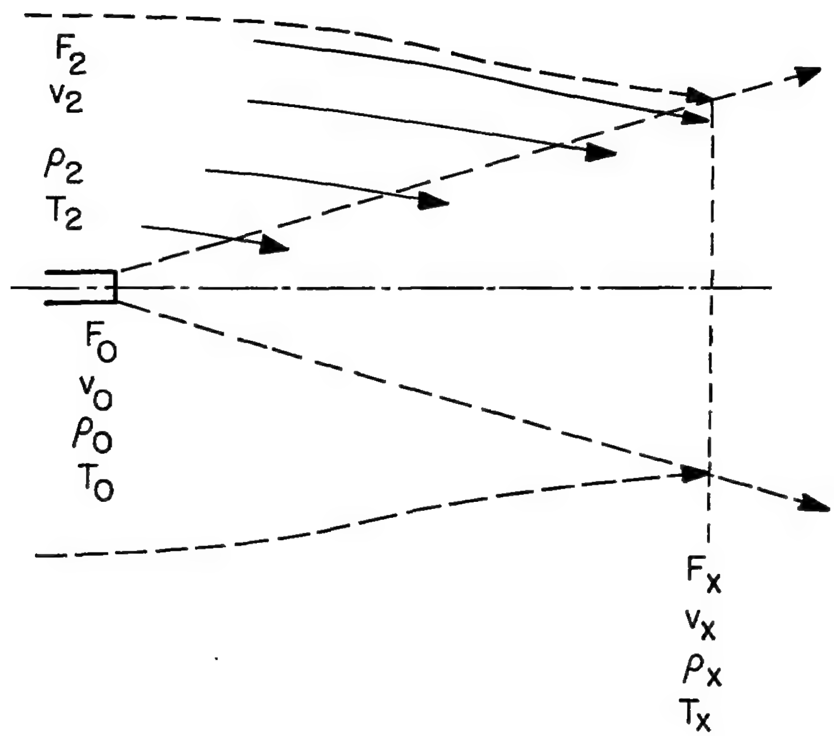

Figure 1.- Free mixed jet surrounding air in motion.

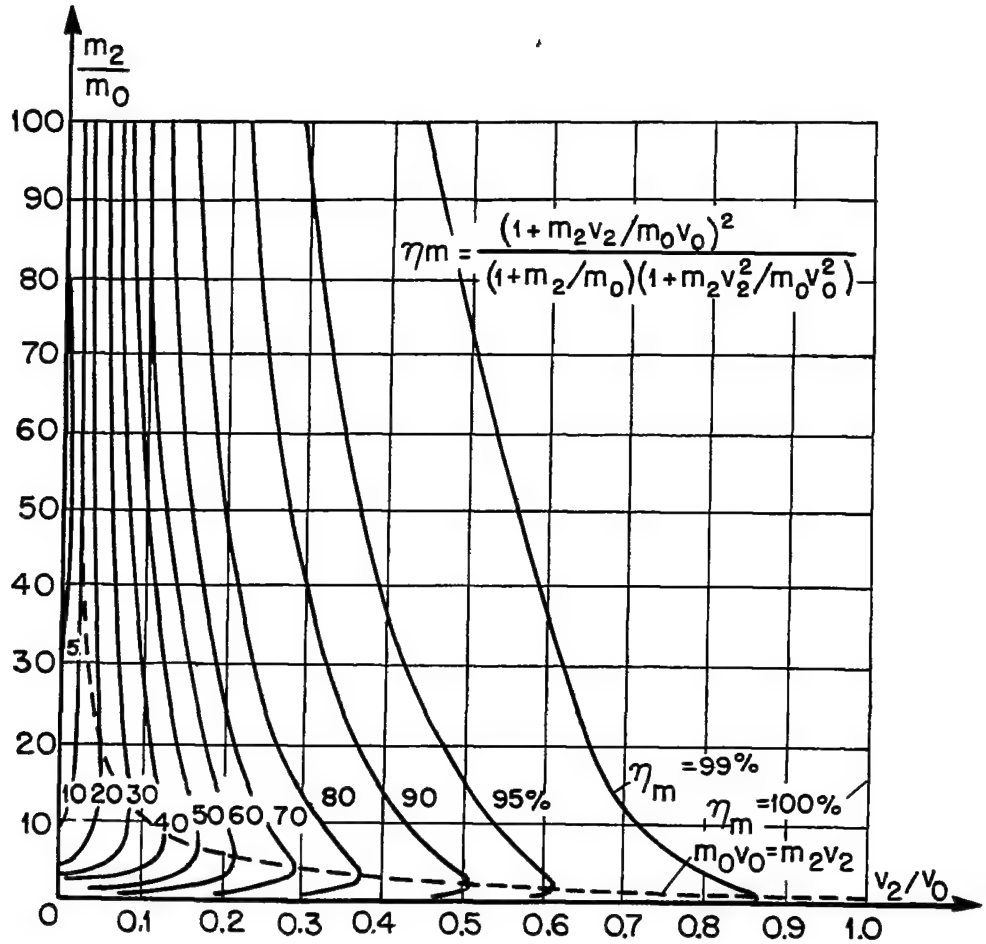

Figure 2.- Propulsive mixing efficiency of constant-pressure mixing.

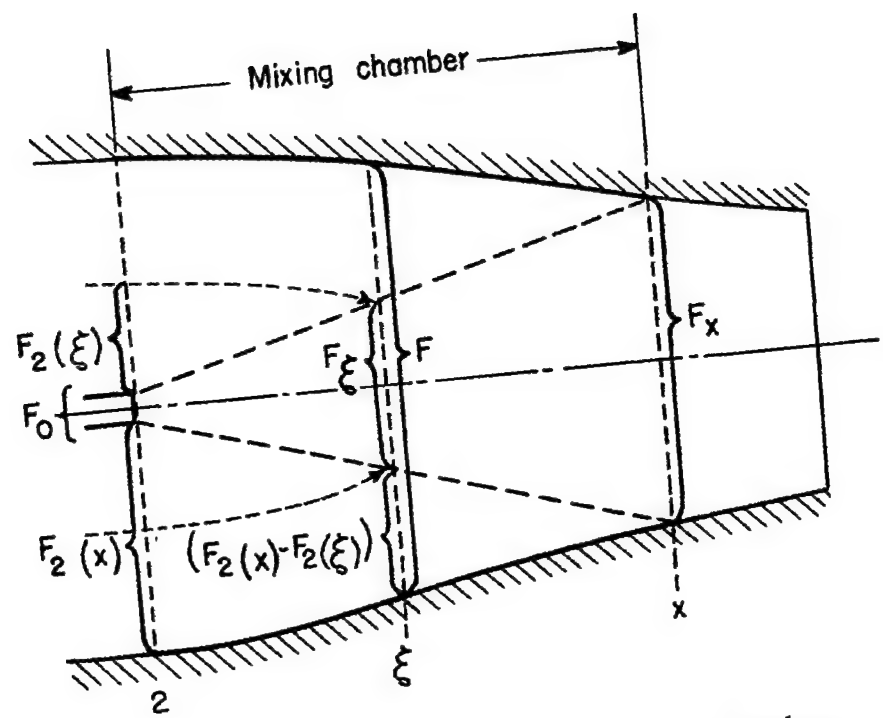

Figure 3.- Constant-pressure mixing chamber.

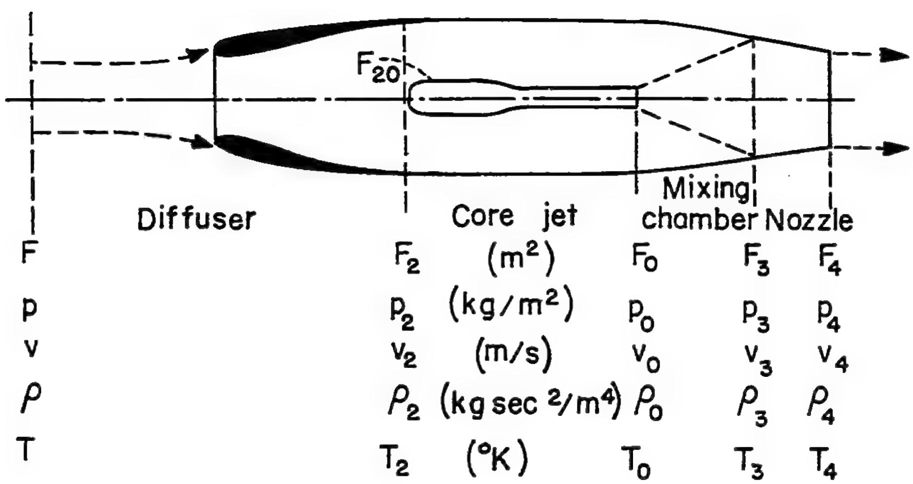

Figure 4.- Ram-jet power plant with momentum heating.

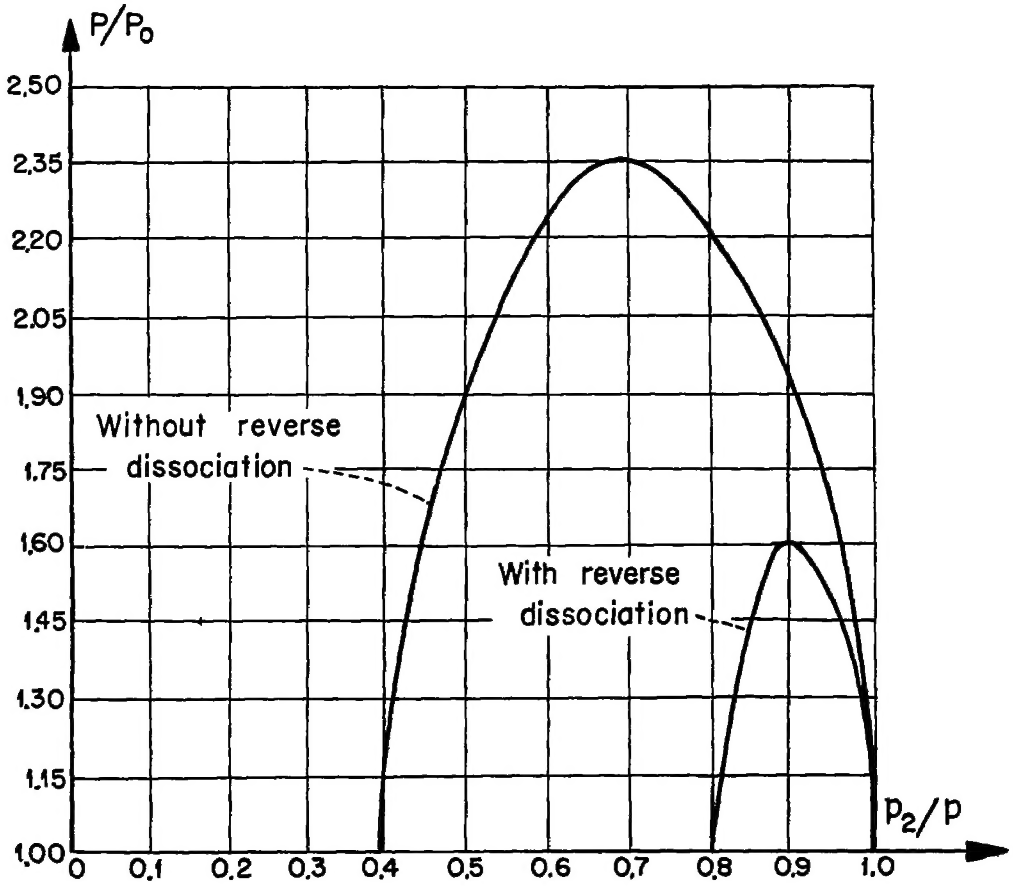

Figure 5.- Dependence of the thrust quotient P/P_0 of a low-pressure mixing chamber in the static case on the mixing chamber pressure with and without reverse dissociation of the rocket jet at 1750° K.

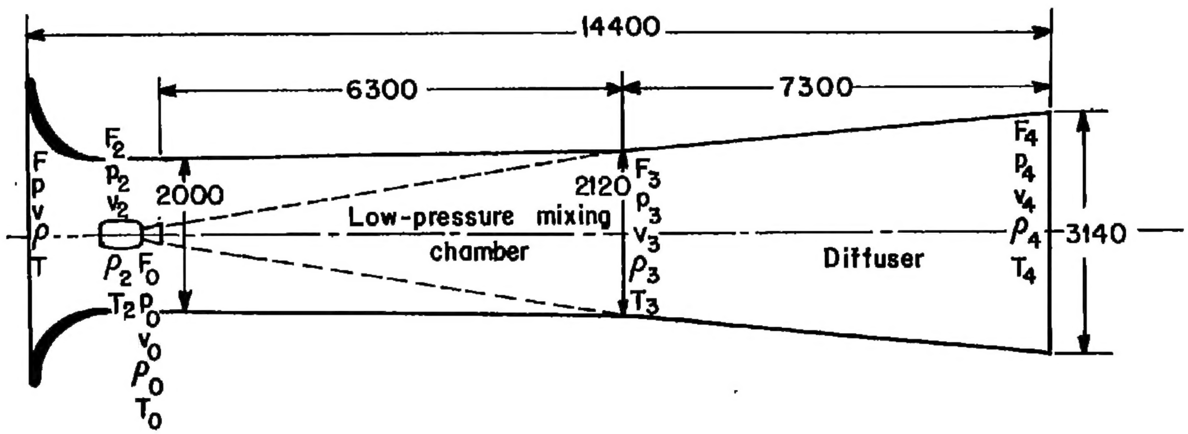

Figure 6.- Low-pressure shrouding of a 3.2-ton rocket for the flight speed $v = 0$ km/h and with a thrust-increase factor of $P/P_0 = 2.36$ for $p_2/p = 0.7$ (high-speed full-scale wind tunnel with $v_2 = 238$ m/s).

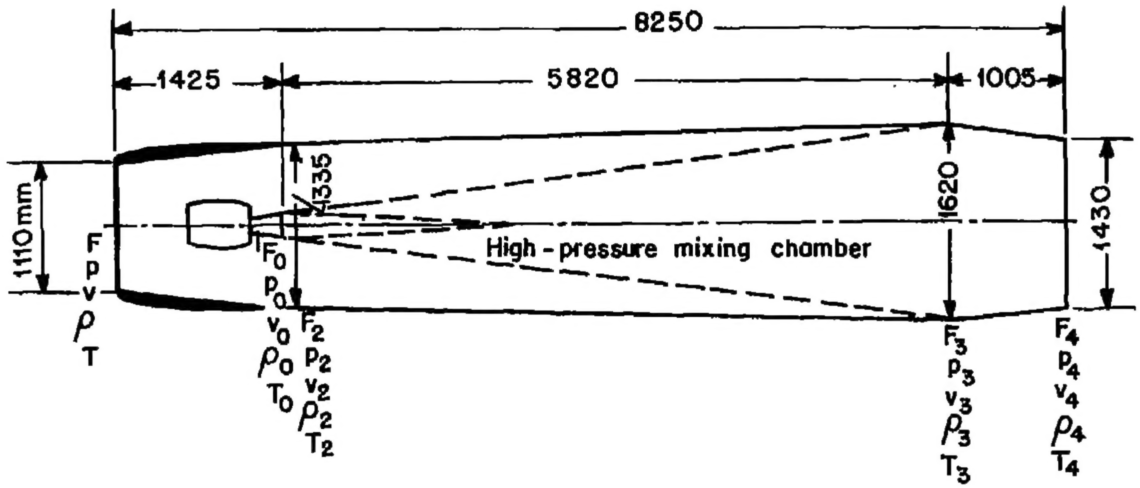

Figure 7.- Theoretically optimum shrouding of a 3.2-ton rocket with dissociated exhaust gas jet for the flight speed $v = 800$ km/h and with a thrust-increase factor of $P/P_0 = 1.195$ (total thrust 3.28 tons, mixing chamber pressure $p_2 = 1.2p$, mass ratio $m_0/m_2 = 1:16.9$).

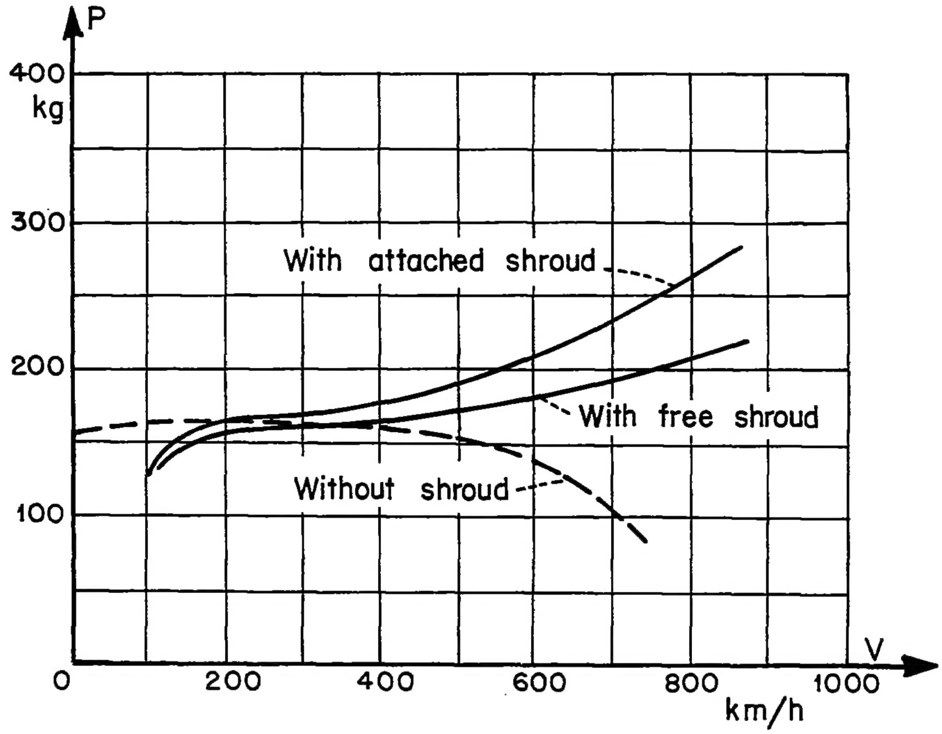

Figure 8.- Thrust variation of a pulse jet tube with and without shroud.